X.

An Account *of a* Method *for extending a* Geographical Survey *across the* Peninsula *of* India.

By Brigade Major Lambton.

Communicated by permission of the Right Honourable the Governor of Fort St. George, in Council.

Having long reflected on the great advantage to general geography that would be derived from extending a survey across the peninsula of India, for the purpose of determining the positions of the principal geographical points; and seeing that, by the success of the British arms during the late glorious campaign, a district of country is acquired, which not only opens a free communication with the *Malabar* coast, but from its nature affords a most admirable means of connecting that with the coast of *Coromandel* by an uninterrupted series of triangles, and of continuing that series to an almost unlimited extent in every other direction; I was induced to communicate my ideas to the right honourable the Governor in Council at *Madras*, who has since been pleased to appoint me to conduct that service, and has supported me with a liberality by which alone it could be carried into execution.

It is scarcely necessary to say, what the advantage will be of ascertaining the great geographical features of a country upon correct mathematical principles; for then after surveys of different districts have been made, in the usual mode, they can be combined into one general map. One surveyor is employed in a district at *Sera*; and another in the

the district of *Chittledroog*. They both have a reference to those particular stations, and their surveys, with respect to them, may be relatively correct: and if *Sera* and *Chittledroog* be laid down right, their respective surveys will fall into their right places on the globe.

It will be unnecessary to state to the Society the imperfect methods that have generally been practised by supposing the earth to be a flat; and yet it has been on this supposition that surveys have been made in general, and corrected by astronomical observation. But although that method of correction may answer for determining the position of places at a great distance, where an error of five or six minutes will be of no very great consequence, yet in laying down the longitudes of places progressively that are not more than twenty miles from one another, it is evident that errors of such a magnitude are not to be overlooked; and an error, even of one mile, would place objects in situations widely different from that which they actually hold on the face of the globe.

If we consider the earth as an exact sphere, we should naturally advert to spherical computation. And having a base actually measured, and reduced to the level, it would be a part of a great circle, while the horizontal angle would be the angle made by two great circles, intersecting each other at the point where the angle was taken. On this hypothesis, the process of extending a survey would be reduced to as great a degree of simplicity as by the method of plane triangles. For then the length of a degree on the meridian could be easily obtained by the celestial arc, and would be equal to a degree in any other direction. The radius of curvature, or the semidiameter of the earth, might also be easily deduced from thence, and being every where the same, the chord of any arc, or the direct distance between two objects subtending that arc, could be computed without the trouble of correcting the observed

served angles. The difference of longitude of any two points might be as easily had; for, knowing the arc between them (which would always correspond with a celestial arc,) and the co-latitudes of the two places, the angle at the pole, or difference of longitude, might be found.

But since the earth is not a sphere, but an oblate spheroid, and differing considerably from a sphere, it becomes necessary to determine the length of a degree on the meridian, and a degree at right angles to that meridian, making the point of intersection of the meridian and its perpendicular the middle point of each degree. Now, in determining the measure of those degrees, if the first measurement, or base line, cannot be had in the meridian, two other objects must be chosen therein, and their distance computed trigonometrically, and then compared with the celestial arc. But here the operations, for obtaining this distance, will be attended with some trouble, on account of its being necessary to calculate the chords of the arcs, and the difficulty of determining the angles made by these chords to a sufficient degree of accuracy. For here we are obliged to assume data, and proceed by an approximating method. And, 1st, we must either suppose the earth to be a sphere, and by taking the three angles made by the intersections of three great circles of that sphere, find the sides in degrees and minutes: then take double the sines of half the arcs, or the chords, and there will be had the three sides of a plane triangle, defined in parts of the radius. With these three sides determine the three angles, and these are the angles for calculating the direct distances. Hence, by knowing the base in fathoms, the chord subtending that base (or arc) may also be had in fathoms, by computing from the radius of the assumed sphere, which we must suppose to be of some given magnitude. Then having the length of the chord in fathoms, and the angles corrected as above,

above, the other chords can be obtained in fathoms also.

Or 2d, Since the chords of small arcs differ very little from those arcs, it will be better to find the distance of the objects from one another by plane trigonometry, the base being one distance. Then we must suppose the earth to be an ellipsoid, whose two diameters have to each other a given ratio. From that, and taking a degree on the meridian to be unity, the ratio of that degree, to a degree in any given direction with the meridian, may be had, as will be shewn hereafter: and that ratio will enable us to allow the appropriate number of degrees and minutes to the computed sides of the triangle, which may then be considered as a spherical one, but whose sides are arcs of circles, having evidently different *radii* of curvature. It is with these arcs, and the observed angles, from which the angles made by the chords are to be obtained. M. De Lambre has given a formula for determining the angles made by the chords of two arcs under these circumstances, having the arcs themselves and the horizontal angle given. The formula is as follows: Let $A =$ angle made by the chords: $a =$ the horizontal or observed angle; D and d the arcs, in degrees, minutes, &c. Then if $x =$ the correction to be applied to the horizontal angle, A will be equal $a+x$. And the first approximate value of $x = -\frac{1}{2}$ tan. $\frac{1}{2} a$. v. s. $(D+d.)$ The second approximate value $= -(\frac{1}{2}$ tan. $\frac{1}{2} a$. v. s. $\frac{1}{2} (D+d) - \frac{1}{2}$ cot. $\frac{1}{2} a$. v. s. $\frac{1}{2} (D-d))$ which is sufficiently near for this purpose; whence $A = a - (\frac{1}{2}$ tan. $\frac{1}{2} a$. v. s. $\frac{1}{2} (D+d) - \frac{1}{2}$ cot. $\frac{1}{2} a$. v. s. $\frac{1}{2} (D-d))$. And if greater exactness be required, it will be $A = a -$ $(\frac{1}{2}$ tan. $\frac{1}{2} a$. v. s. $\frac{1}{2} \overline{D+d} - \frac{1}{2}$ cot. $\frac{1}{2} a$. v. s. $\frac{1}{2} D-d) -$ v. s. x. cot. a. Where x is $= -(\frac{1}{2}$ tan. $\frac{1}{2} a$. v. s. $\frac{1}{2} \overline{D+d} - \frac{1}{2}$ cot. $\frac{1}{2} a$. v. s. $\frac{1}{2} D-d)$, its second approximate value.—And the last term will change its sign to affirmative, if a be greater than $90°$. A demonstration

stration of the above formula has been given by the Astronomer Royal, and may be seen in the Phil. Transactions for the year 1797, p. 450.

Having, by this method, got the angles made by the chords to very near the truth, the rest, with respect to distances, is evident. For the chord of the measured arc (or base) may be had, since by computing the lengths of arcs in any direction, on the ellipsoid, the radius of curvature of that arc is likewise had, and thence the chord. And that chord forms the side of a plane triangle, from which, and the corrected angles, all the data may be had for proceeding upon each of the sides of the first plane triangle.

Now, to determine any portion of a degree on the earth's surface in the meridian, two points may be taken therein, and the direct distance between them ascertained by the above method. Then, by taking the zenith distance of a known star, when passing the meridian, at each extremity of the distance, the celestial arc becomes known in degrees, minutes, &c. from which the terrestrial arc between the two objects is had in degrees, minutes, &c. also:—and having determined the chord in fathoms, the arc may likewise be determined in fathoms, which being compared with the degrees, minutes, &c. the value of a degree is thereby obtained in fathoms.

The length of a degree, at right angles to the meridian, is also easily known by spherical computation, having the latitude of the point of intersection, and the latitude of an object any where in a direction perpendicular to the meridian at that point. For then the arc between these two points, and the two celestial arcs or colatitudes, will form a right angled triangle, two sides of which are given to find the third, which is the arc in question. And this will apply either to the sphere or spheroid. That arc being known, in degrees and minutes, and the chord

chord having been previously determined in fathoms, being a side of one of those plane triangles, formed by the chords of the terrestrial arcs; the length of that arc can also be determined in fathoms; and, therefore, a degree may be determined in fathoms, having its middle point the point of intersection with the meridian.

THUS having obtained the length of a degree upon the meridian, and its perpendicular, in any given latitude, they will serve as *data* for computing the latitude and longitude of places near that parallel, and near to that, or a known meridian, by means of the chord of a terrestrial arc, oblique to the meridian and its perpendicular, and the chord of the meridional arc intercepted by a great circle falling from the extremity of the oblique chord, and cutting the meridian at right angles. For it will be easy to find the measure either of the part of the meridian, or the portion of the circle at right angles thereto (even by using the observed angles;) and if these be converted into degrees, minutes, &c. according to the length of a degree upon the respective circles, the former will give the difference of latitude, and consequently, by addition or subtraction, the real latitude: the latter, with the co-latitude thus obtained, will enable us to find the angle at the pole. In both these cases the truth may be obtained to within one-fourth, and generally one-tenth of a second, (limiting the operations to a certain extent from a known parallel and meridian;) and that without having recourse to observation, or depending on any hypothesis of the earth's figure.

IT will readily occur to the reader, that had the ratio of the assumed diameters been what it really is, and supposing the earth to be an exact ellipsoid, the computed and measured degrees ought to come out the same. But the reason for computing the length of ellipsoidal arcs was only to gain the approximate values of the angles made by the chords, by doing which, we can come nearer the truth, than by supposing them to be spherical; and though these arcs may not be precisely correct, yet it has been found that a trifling deviation from the truth will not sensibly affect the angles.

It may be further observed, that we are not certain, either of the ratio of the earth's diameters, or of its being an ellipsoid. We have assumed that figure, and have drawn our results from the average of different measurements, made in different parallels, though among themselves they appear contradictory: but we must adopt them, until better measurements can be made, to enable us to come nearer the truth. Should the figure of the earth prove to be the ellipsoid, and the ratio of the equatorial diameter to the polar axis become known, a celestial arc would afford a datum in any assigned latitude, by which, and the observed angles corrected, the direct distances might be computed, and also the distance of any object from a known meridian and its perpendicular, and consequently its longitude and latitude.

But should the earth prove to be neither an ellipsoid, nor a figure generated by any particular curve, of known properties, but a figure whose meridional section is bounded by no law of curvature, then we can obtain nothing until we have an actual measurement, to be applied as has been already mentioned.

Thus much I have thought necessary to premise, that the general principles of the work I have before me may be understood;—principles, which I believe have never been applied in Indian geography, though in England sufficient has been done to manifest their perfection, and to give those gentlemen, who have applied them, a distinguished reputation in the annals of science: and I own, that it was from reading the details of their operations I was first led to consider the subject. The publications of the late

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GEN. ROY, relative to his measurements on *Hounslow-heath* and *Rumney-marsh*, with his continuations of triangles;—and the later accounts of a trigonometrical survey along the southern and eastern coasts of *England*, by LIEUT. COL. WILLIAMS, CAPT. MUDGE, and MR. DALBY, are works which I consider as a treasure.

WITH respect to the plan of my operations, had I been possessed of an instrument, which I could have thought sufficiently accurate for taking horizontal angles, I should have measured a base somewhere near the eastern coast, both on account of its being a more regular country, and nearer the level of the sea, to which all future measurements and distances must be reduced, and because I could have computed my longitude from the *Madras* observatory. There would have been, besides, some probability of getting a measurement in the meridian, or so near it, that all oblique directions might have been accurately reduced to it, and that would be a means of at once obtaining the length of a degree on the meridian: and as a degree has never yet been measured in this parallel, it is no trifling circumstance to look forward to, because we should get a datum in the first instance, for computing the ratio of the earth's diameters, considering it to be an ellipsoid. And as I have the same kind of chain, made by the same incomparable artist, Mr. RAMSDEN, as that with which COLONEL WILLIAMS and CAPTAIN MUDGE measured their bases; from a comparison between two measurements made in parallels so distant from each other, with instruments of the same kind, and reduced to the same standard temperature; there is some reason to hope that computations made from such measurements may come nearer the truth than any other.

HOWEVER, this is an object to which I look forward when those instruments arrive, which government

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ment has been pleased to authorise me to send for. At present it seemed most desirable that I should begin in *Mysore*, and endeavour to forward the surveys of that country. Having made a first measurement there, I think, with the instruments I at present possess, it will be best not to extend my operations too far from some assumed meridian, as I can depend more upon meridional celestial arcs than upon any computed oblique ones. The instrument I have for taking zenith distances is a zenith sector of five feet radius, made by Mr. RAMSDEN, with a micrometer scale that defines nearly one-tenth of a second. With this I can determine two parallels of latitude to be depended on between which to compute by terrestrial measure the relative situations of intermediate places as to latitude. The instrument with which I take horizontal angles is a circular transit instrument, made by Mr. TROUGHTON, whose horizontal limb is only eight inches radius, without a micrometer, but which is graduated to 10″; and though it is an excellent instrument, correct and easy in its adjustments, yet its powers are not sufficient for taking horizontal angles where they are to be reduced to the angles made by the chords.

SECTION I.

Containing an Account of the Measurement of a Base Line on the Table Land of the MYSORE *Country near* BANGALORE.

I MENTIONED above my reasons for making a measurement in the *Mysore* country. This measurement may, however, not be thought so satisfactory as if it had been done near the sea coast, on account of of not being certain as to the exact height above the level of the sea, since that height was determined by corresponding barometrical observations made at *Madras*, and at each extremity of the base, and I am well aware that those results will be exceptionable. But I was careful to found my computations on those observations only which were made when a perfect uniformity in the state of the atmosphere had existed for several days together; that is, when the barometer and thermometer at each place, and at the same hour of the day, had suffered scarcely any sensible variation for a considerable time. And since the quantity to be deducted from the base on account of the height is little more than 8, 5 feet, upon the whole, any error that might arise in correcting for the temperature and density of the atmosphere would be but trifling; I shall therefore, for the present, rest satisfied until the height can be determined trigonometrically, and proceed to give an account of the operations of the measurement, and of the apparatus made use of.

CHAIN.

The chain is of blistered steel, constructed by Mr. Ramsden, and is precisely alike, in every respect, with that used by General Roy in measuring his base of verification on *Rumney marsh*. It consists of 40 links of 2½ feet each, measuring in the whole 100 feet. It has two brass register heads, with a scale of six inches to each; these scales slide in the brass heads, and are moved by a finger screw, for the purpose of adjusting exactly the two extremities of the chain when extended: in short, every part of it is the same as the one above mentioned, which has been fully described in the Philosophical Transactions of 1790, and therefore it is unnecessary to say more on the construction of that instrument here.

It appears from the best information I have respecting it, that it was measured off by the brass standard when the thermometer stood at 62°, and was, in that temperature, exactly 100 feet in length.

From the want of a proper standard scale and beam compasses, I would not undertake to determine its length, compared with brass; because I did not think that laying off any determined number of feet from the sliders in the register heads, and by a pair of common compasses, could be done with sufficient accuracy, so as to enable me to find out at *what* degree of temperature the chain had measured 100 feet by the brass scale. And as I had been informed by Doctor Dinwiddie, from whom it was purchased, that, to the best of his recollection, it had been adjusted to 100 feet at the standard temperature of 62°; I therefore rested satisfied until further information may be obtained respecting it; and it is probable, that any correction on account of temperature, will not amount to more than two or three feet, and an error of that magnitude in a length of near 7½ miles cannot be of very great moment in geography, which is the principal object at present.

There is another circumstance it may be necessary to mention with respect to the chain. From the same want of a standard measure, I have not attempted to determine its wear; but I observe that in the measurement of the base of verification on *Salisbury* plains, the chain used there was very little affected by being in use about seven weeks. And in order to prevent the wear as much as possible, I allotted twenty coolies, that is one to every two links, whose sole business it was to lift out the chain and lay it on the ground whilst the coffers were moved forward, and then to replace it when they were ready. All this was done with the greatest care, and always by the word given them, that the motion might be as trifling as possible. This mode was practised during the whole measurement, so that I am in

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hopes no very serious error can arise from the wear of the chain.

COFFERS.

Those were of twenty feet each in length, six inches wide in the middle, three at the extremities, and about four inches deep; the sides were near seven inches, and passed below the bottom two inches—they were not of the dimensions of those of General Roy, on account of the difficulty of procuring boards for the purpose. The same difficulty obliged me to be satisfied with five in place of fifteen; but as I had a great number of people with me, I apprehended no great difficulty in taking out the chain and laying it on the ground while the coffers were moved forward.

PICKETS.

Twelve strong pickets of three inches diameter, hooped and shod with iron, were made use of—they were of different lengths, from three to four feet; on the top of each picket was placed a piece of very hard seasoned wood, eight inches in length and four in breadth, on the under side of which was fixed with two screws, a hoop of iron, fitted to receive the one on the picket, and to screw firmly upon it by a small screw on the side, when placed properly in the line. This simple contrivance seems to answer the intended purpose for receiving and supporting the ends of the coffers; the two pickets on which the brass register heads were placed, are in all respects the same as those described by General Roy. There is also the same apparatus for the drawing post and weight post, only in place of the iron ferrule, the brass clamp and pulley are fixed upon pieces of very

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hard well-seasoned wood, in a manner so simple as to render a description unnecessary.

I FOUND, however, in the course of practice, that tripods, with elevating screws in the centre, answered much better than the pickets for the intermediate ends of the coffers, particularly as a very great part of the ground was hard and stony. Those tripods are described by GENERAL ROY. Those which I used, as I had not the means of getting better, were no more than the common wooden press screw, made to move up and down by a female screw with handles; the top of the tripod being a thick piece of wood for the screw to pass through, with another piece of wood three or four inches below that to keep it steady—but a boxed tube to receive the screw is to be preferred.

BONING TELESCOPE.

FOR the purpose of fixing the objects in allignement, I used the circular transit instrument, which answers remarkably well, both for that purpose and for laying off the principal elevations and depressions of the different hypothenuses; but when the pickets are to be placed so that the coffers may be laid in the line of the hypothenuse, I made use of one of Mr. RAMSDEN's spirit levels; but in place of using its three legs, I took them off and placed the telescope, with its adjusting screws, upon a tripod, having an elevating screw in the centre, passing through a tube with a small iron screw to keep it firm. On the top of this elevating screw was fixed a piece of board about ten inches square—upon that again was placed another piece, which was made to move in a groove by a finger screw, and upon this moveable piece the levelling telescope, with its apparatus, was fixed, having its axis at right angles to the direction of the groove, so that by the finger screw it could easily be moved to the right or left, and brought into the direction of the allignement.

A SMALL

TABLE containing t 2° 54′ 64″ N. and extending 7,4321 miles
N. Easterly, making an uses, or measured distances. The Second,
the length of each in fe with the horizon. The Fourth, the hori-
zontal oblique angles. ce them to the horizon. The Sixth, the
quantities to be subtract he line. The Seventh, the perpendicular
ascents and descents to above or below the termination of the one
preceding; and the Nin

No. of the Hyp.	Length of each in Feet.	Elev	ment from aft. Below Inches.	Mean of five Thermometers.	REMARKS.
		o 7			
1	1100			85,10	Commenced on the 14th October 1800.
2	1100			80,33	
3	500			81,26	
4	400			80,60	
5	100		5,	83,20	
6	200		11,2	81,	
7	900	o	7,3	81,93	
8	500		8,2	79,98	
9	300		6,2	82,20	
10	400		5,0	79,22	
11	300		11,1	75,66	Computed from a measured base of 200 feet—this was done to avoid a deep gully and some rocky ground.
12	312,422			79,05	
13	300			79,96	
14	200		17,5	84,80	
15	400	2 3		81,05	The 2d chain of this hypothenuse extended across the *Bangalore* road.
16	500	1 5	6,9	83,72	
17	400	1 2	2,0	81,12	
18	300	3	10,4	87,80	
19	1000		9,2	85,11	
20	400		8,6	85,20	
21	400		7,8	85,37	
22	400		10,4	82,	
23	600		6,2	88,05	
24	500		2,1	86,74	
25	200	o 1	6	85,75	
26	300	1 4	5,6	79,83	
27	200	2	0,0	88,40	
28	100	1 4	5,3	80,60	
29	400	5	3,0	80,70	
30	300			87,60	
31	400		8,2	81,05	
32	200		18,1	75,	Computed from a horizontal base of 5 chains. The angles were taken with the greatest care by the circular instrument—this was necessary to avoid a small tank which was dry when the ground was first inspected.
33	1308,08364		4,6	75,40	
34	500	1 3		71,62	
35	200	4	6,4	80,50	
36	300			80,63	
37	300		16,9	84,90	
38	200		9,5	81,45	
39	400		7,8	79,52	
40	300			87,36	
41	900			83,24	
42	500		7,6	79,06	
43	800		10,2	84,79	
44	800			91,11	
45	800		8,7	89,31	

Perpendicular.		Commencement from the last.		Mean of five Thermometers.	REMARKS.
ts.	Descents.	Above Inches.	Below Inches.		
t.	Feet.				
	10,34085		11,4	89,54	
9089		10,1		90,22	
				90,50	
	3,19978		1,0	8: 80	
	3,49060		3,3	78,12	
		4,2		84,56	
			10,1	83,80	
			29	85 20	
			7,4	79 66	
9124		6,2		79,97	
11465		15,2		85,50	
27190			18	80,70	
98159			13 5	88,08	
			8,5	93,20	
	2,79252		8	80.75	The oblique direction was taken to avoid the corner of the small village of *Nagasundrum*.
	4.45044		14,9	87,43	
	11,86784		12,9	91,27	
	5,58488		13,0	83,75	
			16,4	90,64	This hypothenuse is computed, but may be measured during the dry season. From the excessive rains that had fallen, the arm of a large tank had extended a considerable way across the line—the angles for computing this distance, as well as those of the oblique directions, were the mean results of three different observations with the circular instrument; the base was a level of 5 chains.
19096				89,00	
83103				88,80	
32996			7,5	87,20	
94304			1,5	85,47	
99967				79,14	
91985			8,8	86,47	
	1,45440	0,9		76,45	
36168			8,4	84,70	
57378			5,1	87,90	
88644			9,6	70.90	
97236			1,2	78.20	
11908			2.3	86,65	
98614			7.8	00,05	
55656			23,2	86.35	
20631		5,2		81,60	
		6,1		83,20	
				84,84	Computed from a base of 5 chains.
59425			102,4	85,50	
	0,34908	1 0,2		11,70	
	1,04720		10	77,20	
55706			9,8	76,96	
51096		25,5		85,00	
37696			1,3	86,70	
46852			9,9	70,64	
05112			8,1	86,07	Completed on the 10th December.
3 622			5.2	88,40	
54433	241,34419	140,0	631,5	83,5	

Feet.
\- - - - - - - - 39332,82212
\- - - - - - - - — 5,41273
\- - - - - - - - — 59,30097

f 62° and the mean temperature of measurement being 83,5 very nearly, the

add - - - - - - - - - +5.14473
\- - - - - - - - - 39273,253
will be - - - - - - - - 39267,706

d brass scale, there is a coincidence of measure; that is, that 100 feet of steel shall coincide
he *temperature of coincidence*.

A SMALL square picket, or boning rod, with a piece ten inches in length, fixed at right angles, and made to slide up and down, and fasten by a small screw, was placed at the further extremity of the hypothenuse, and the sliding piece put at a convenient height: that piece therefore marked the angle of elevation or depression. The height of the axis of the transit circle, (when that instrument was used,) having been taken by a plumb line, as well as the point directly under its centre: Then having marked out one hundred feet, by a common measure, exactly in the allignement, I removed the transit, and placed the tripod, with its apparatus, precisely on the spot which marked its centre; and measured its height above that spot, comparing the centre, on which the levelling telescope moves, with the transverse axis of the transit, (having previously determined the most convenient height for the coffers to be from the ground.) Then I took the exact measure of the space between the axis of the transit and that of the levelling telescope, and applied it to the boning rod at the extremity of the hypothenuse, and made a mark, at that distance, below the cross slider.

THE level was then adjusted by the screws and spirit level, and its centre brought into the allignement; which being done, the axis of the telescope was elevated, or depressed, until the cross wire corresponded with the mark on the boning rod.

IF the angle of the hypothenuse be beyond the limits of the vertical screw of the level, the tripod must incline so as to bring it within those limits, and that angle of inclination noticed, that the perpendicular height may be justly determined; that however never happened.

BUT, as the angles of elevation and depression were in general very small, I contrived to take them with a small sextant, both on account of saving time, and to avoid running unnecessary risk with the circular instrument. The method which I used was as follows:

I FIRST laid out the direction of the hypothenuse, by a boning rod, placed at a distance, to be seen with the small telescope of the sextant. Another boning rod was then placed at a convenient distance, so that the cross vane might be brought to correspond with the cross wires of the levelling telescope, after it had been carefully adjusted to the horizontal direction by the spirit level. Then, upon the same boning rod was placed another cross vane, and the telescope elevated, or depressed, by the finger screw, until the cross wires were brought into the direction of the hypothenuse by the vane on the distant boning rod.—In taking the angle with the sextant, I placed the axis of motion close to the Y of the levelling telescope, at the opposite end, with the finger screw; so that the two vanes, on the distant and near boning rods, appeared to correspond in the reflector of the sextant, and then the angle was taken.

IN this manner all the smaller angles of elevation and depression were taken, and though not exactly in the way I could have wished, yet I have no doubt of their being nearly correct, perhaps as much so as any direction can be measured.

HENCE the line was determined, which passed through the axis of the levelling telescope, and was parallel to the hypothenuse. In order to place the pickets for receiving the coffers, a piece of wood was contrived for being placed upon the head of each, with a cross vane to slide up and down. Then, a picket was driven, at any given distance in the allignement, and the above piece applied to its top. When the cross piece corresponded with the mark, the picket remained in that state, and the rest of them were driven down in the same manner, and the piece applied to their respective heads; and being all adjusted by that means, their tops were consequently parallel to the line of direction.

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The coffers were then put upon the pickets, and having all their bottoms of the same thickness, they therefore formed the plane in which the chain was to be extended.

When any hypothenuse was terminated, a line, with a plummet, was let fall from the arrow upon the feather edge of the chain; and the point on the ground was marked, which was defined by the point of the plummet, (for a brass register head was there unnecessary,) and the height of that extremity of the chain, from the ground, was carefully taken. The new hypothenuse, therefore, commenced from that same point, and the arrow at the beginning of the next chain was made to coincide with a plumb line falling to the said point. And the height also of that end of the chain, from the ground, was taken; by which means, the ascent or descent of the commencement of the new hypothenuse was determined.

When the chain was extended in the coffers, it was fixed at one end to the drawing post, and from the other an 8½ inch shell was suspended. The leading register head was then brought by the finger screw, so that some division might correspond with the arrow. Five thermometers were then put into the coffers, (one into each,) and there remained for some minutes, a cloth at the same time covering them. They were then taken out, and the mean temperature marked down. This was done to every chain, and a mean of each hypothenuse was afterwards taken, and the result served to determine the equation arising from expansion and contraction, for correcting the whole apparent length of the base.

Every thing having been prepared, the measurement commenced on the 14th October, and was completed on the 10th December: the particulars thereof will appear in the following table.

Observations for the Latitude of the southern extremity of the Base, and the Meridian at that point.

For the meridian, I observed the angle which the line made with the polar star when at its greatest western elongation; and computed its azimuth, at that time, from having the latitude of the place, and the apparent polar distance given—at that season of the year a double azimuth could not be taken in the night time, and my telescope had not sufficient powers to observe the star in the day time.

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Now, since the expansion of brass is different from that of steel; it follows, that when the measurement is made in a higher or lower temperature than that in which the steel and brass coincided, there will be an equation; which must be applied to the apparent measure of the chain, in order to bring it to the brass measure. I shall call this higher or lower temperature, the *temperature of measurement.*

After the steel chain has been reduced to brass measure, it may be found necessary to reduce the brass standard itself, to the *space* it would have measured, or extended over, in a higher or lower temperature. Let that be called the *standard temperature.* Now upon a slight examination of these, it appears that they will resolve themselves into three cases.

Case 1st. When the standard temperature and the temperature of measurement are both *above* the temperature of coincidence.

Let the brass standard and steel chain coincide, when the thermometer is at 54°; and let a space be measured by the chain at the temperature of n degrees, so that $\overline{n-54^\circ}$ shall express the number of degrees above the temperature of coincidence, when the measurement is made. Now, the length of the chain at 54° was precisely a given number of feet, (we will suppose 100 feet,) by the brass scale. And since ,00763 inches is the expansion of 100 feet of steel for one degree of the thermometer, it follows, that when the chain is applied at the temperature of n° it will extend *over a space on the ground* equal to

$$100+\frac{\overline{n-54^\circ}\times,00763}{12}$$ feet, if measured by the brass scale in the temperature of 54°.

So far as to the temperature of 54° when the brass and steel coincide; that is, when 100 feet of brass coincide with 100 feet of steel at that degree of temperature. But suppose it should be thought necessary to change the standard temperature to n°, the temperature of coincidence being still at 54°:—that is to say, let the space above-mentioned be measured by the brass standard at the same temperature n° as when the chain

No. of the Hyp.	Length of each in Feet.	Angles of Elevat. ° ′ ″	Angles of Depr. ° ′ ″	Oblique horl. angles with the line. To the left.	Oblique horl. angles with the line. To the right.	Deductions from each Hypothenuse.	Deductions from oblique Directions.	Perpendicular. Ascents.	Perpendicular. Descents.	Commencement from the last. Above Inches	Commencement from the last. Below Inches	Mean of five Thermometers.	REMARKS.
								Feet.	Feet.				
46	900		39 30			,05960			10,34085		11,4	89,54	
47	400	2 30				,00009		0,29089		10,1		90,22	
48	200											90,50	
49	400		27 30			,01282			3,19973		1,0	82,80	
50	400		30			,01524			3,49060		3,3	78,12	
51	200									4,2		84,56	
52	200				9 31 44						10,1	83,80	
53	100										29	85,20	
54	300										7,4	79,66	
55	400	1 19				,10560		9,19124		6,2		79,97	
56	500	1 46				,23765		15,41465		15,2		85,50	
57	500	0 50				,05290		7,27190			18	80,70	
58	500	0 20 30				,00889		2,98159			13,5	88,08	
59	200						37,24871				8,5	93,20	
60	400		0 24	5 37 16		,00976			2,79252		8	80,75	The oblique direction was taken to avoid the corner of the small village of *Nagasundram*.
61	300		51			,03300			4,45044		14,9	87,43	
62	800		51			,08800			11,86784		12,9	91,27	
63	400		48			,03900			5,58488		13,0	83,75	
64	2097,21048										16,4	90,64	This hypothenuse is computed, but may be measured during the dry season. From the excessive rains that had fallen the arm of a large tank had extended a considerable way across the line—the angles for computing this distance, as well as those of the oblique directions, were the mean results of three different observations with the circular instrument: the base was a level of 5 chains.
65	600	1 11				,12798	22,05226	12,39096				89,00	
66	400	1 24 30				,12082		9,83103				88,80	
67	400	1 3				,06716		7,32996			7,5	87,20	
68	400	48 30				,03818		5,64304			1,5	85,47	
69	500	27 30				,01600		3,99967				79,14	
70	300	22				,00615		1,91985			8,8	86,47	
71	400		12 30			,00264			1,45440	0,9		76,45	
72	400	16				,00432		1,86168			8,4	84,70	
73	400	56 30				,05400		6,57378			5,1	87,90	
74	200	1 24				,05970		4,88644			9,6	79,90	
75	400	35				,02072		4,07236			1,2	78,20	
76	200	1 28				,06552		5,11908			2,3	86,65	
77	400	1 9 30				,08173		8,08614			7,8	90,05	
78	400	1 23				,11656		9,65656			23,2	86,35	
79	200	1 29 30				,06777		5,20631		5,2		81,60	
80	100									6,1		83,20	
81	615,106											84,84	Computed from a base of 5 chains.
82	200	34 30				,01010		1,59425			102,4	85,50	
83	300		4			,00021			0,34908	10,2		81,70	
84	200		18			,00274			1,04720		10	77,20	
85	300	2 1				,18582		10,55706			9,8	76,96	
86	400	1 57				,23164		13,61096		25,5		85,00	
87	400	1 12				,08772		8,37696			1,3	86,70	
88	400	47				,03740		5,46852			9,9	79,64	
89	400	40				,02708		4,65412			8,1	86,07	
90	300	21 30				,00586		1,87622			5,2	88,40	Completed on the 10th December.
[illegible]	39332,82212					5,41273	59,30097	254,54433	241,34419	140,0	63[illegible],5	83,[illegible]	

	Feet.
Apparent length of the base measured and computed 393,3282212 chains equal	39332,82212
Sum of all the deductions in column 5,	— 5,41273
Sum of the deductions in column 6,	— 59,30097

(A) Then if the chain was compared with the brass standard and measured 100 feet at the temperature of 62° and the mean temperature of measurement being 83,5 very nearly, the correction for the chain's expansion will be $= \frac{83,5 - 62^{\circ} \times ,0073}{12} \times 39332,822$ feet nearly, which add — + 5,14473

Therefore the true length of the base in the temperature of 62° will be — 39273,253

Which being reduced to the level of the sea, by allowing the height above *Madras* to be 2901 feet, will be — 39267,706

(A) We will first suppose, that, when a steel chain is measured off, in any given temperature, by the standard brass scale, there is a coincidence of measure; that is, that 100 feet of steel shall coincide with 100 feet of brass. And this temperature being denoted by the degrees on the thermometer, I shall call the *temperature of coincidence*.

TABLE containing the particulars of the meaſurement of a baſe line near *Bangalore*, commencing in latitude 12° 54′ 64″ N. and extending 7.4321 miles N. Eaſterly, making an angle with the meridian 0° 57′ 7″. The 1ſt column contains the number of hypothenuſes, or meaſured diſtances. The 2d the length of each in feet. The 3d the angles of elevation or depreſſion which each hypothenuſe makes with the horizon. The 4th the horizontal oblique angles. The 5th the quantities to be ſubtracted from the reſpective hypothenuſes to reduce them to the horizon. The 6th the quantities to be ſubtracted from the oblique (horizontal) direction to reduce them to the horizontal diſtance in the line. The 7th the perpendicular aſcents and deſcents to each hypothenuſe. The 8th the commencement, in inches, of every hypothenuſe above or below the termination of the one preceding; and the 9th contains the mean temperature during the reſpective meaſurements.

No. of the Hyp.	Length of each in Feet.	Angles of		Oblique hor. angles with the line.		Deductions from each Hypothenuſe.	Deductions from oblique Directions.	Perpendicular.		Commencement from the laſt.		Mean of five Thermometers.	REMARKS.
		Elevat.	Depr.	To the left.	To the right.			Aſcents.	Deſcents.	Above Inches.	Below Inches.		
		° ′ ″	° ′ ″					Feet.	Feet.				
1	1100		1 0 30			,17050			19,35761			85,10	Commenced on the 14th October, 1800.
2	1100		0 38 30			,06900			12,31886	6,0		80,33	
3	500									14,3		81,26	
4	400									10,0		80,60	
5	100										5,	83,20	
6	200		1 9			,04028			4,01358		11,2	81,	
7	900	0 3 40				,00040		,09599			7,3	81,93	
8	500		57 30			,07000			8,17229		8,2	79,98	
9	300		1 10			,06219			6,10824		6,2	82,20	
10	400		1 13			,09020			8,49328		5,0	79,22	
11	300		2 10			,21447			11,34195		11,1	75,66	
12	312,422									1,4		79,95	Computed from a meaſured baſe of 200 feet—This was done to avoid a deep gully and ſome rocky ground.
13	300											79,96	
14	200										17,5	84,80	
15	400	2 35 50				,4'091		18,12582		3,75		81,05	
16	500	1 36				,19495		13,96080			6,9	83,72	The 2d chain of this hypothenuſe extended acroſs the *Bangalore* road.
17	400	1 23				,11656		9.65656			2,0	81,12	
18	300	33				,01353		2,87976			10,4	87,80	
19	1000										9,2	85,11	
20	400		22			,00820			2,55980		3,6	85,20	
21	400		1 11 30			,08646			8,31882		7,8	85.37	
22	400		1 14			,09268			8,60964		10 4	81,	
23	600		1 45			,27984			18,32310		6,2	88,05	
24	500		1 40			,21150			14,54236		2,1	86,74	
25	200	0 10				,00084		0,58178			6	85,75	
26	300	1 40				,12690		8,72541			5,6	79,83	
27	200	2 4				,13010		7,21246		8,5	0,0	88,40	
28	100	1 49				,05026		3.17015			5,3	80,60	
29	400	54				,04936		6,28292			3,0	80,70	
30	300		0 29			,01068			2,53071	2,4		87,60	
31	400		56			,05308			6,51560		8,2	81,05	
32	200		1 19			,05280			4,59562		13,1	75,	
33	1308,08364										4,6	75,40	Computed from a horizontal baſe of 5 chains. The angles were taken with the greateſt care by the circular inſtrument—This was neceſſary to avoid a ſmall tank which was dry when the ground was firſt inſpected.
34	500	1 32 20				,18034		13,42772				71,62	
35	200	44				,01641		2,55974			5,4	80,50	
36	300											80,63	
37	300		41			,02133			3,57783		16,9	84,90	
38	200		1 7 30			,03855			3,83735		9,5	81,45	
39	400		1 38 30			,16418			11,45943		7,8	79,52	
40	300		1			,04569			5,28572			87,36	
41	900											83,24	
42	500		31			,02035			4,50870		7,6	79,06	
43	800		41			,05688			9,54088		10,2	84,79	
44	800		48			,07800			11,16976	10,1		91,11	
45	800		50			,08464			11,63512		8,7	89,31	

THE observations were made on the 3d, 14th, and 21st of December, at which times the apparent azimuths of the star were 1°. 47′. 42″. 1°. 47′. 40+″, and 1°. 47′. 40−″, leaving out the decimals of the seconds; and the mean of the angles made with the line and the star at those times was 2°. 45′, 50″, 2°. 45′. 20″, and 2°. 45′; which, compared with the apparent azimuth, will give a mean of 57′. 40″ nearly N. Easterly, which is the angle made by the line with the meridian.

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chain was extended over that space. Then, if the expansion of brass and steel had been the same, the space which measured $100+\frac{\overline{n-54^\circ}\times ,00763}{12}$ feet by the brass, when the thermometer stood at 54°, will now measure $100+\frac{\overline{n-54^\circ}\times ,00763}{12}-\frac{\overline{n-54^\circ}\times ,00763}{12}$ or 100 feet; by reason of the brass having increased $\frac{\overline{n-54^\circ}\times ,00763}{12}$ feet, in 100 feet. But since 100 feet of brass expands ,01237 inches for one degree of the thermometer, the space over which the steel chain extended at n° will measure by the brass standard

$$100+\frac{\overline{n-54^\circ}\times ,00763-\overline{n-54^\circ}\times ,01237}{12}$$

feet: and, from a parity of reasoning, if n° be not the temperature in which the space is to be measured by the brass standard, but s° which is therefore the standard temperature. Then the measurement reduced to that temperature will give $100+\frac{\overline{n-54^\circ}\times ,00763-\overline{s-54^\circ}\times ,01237}{12}$ feet, if measured at s° of temperature.

CASE 2d. When the standard temperature and the temperature of measurement are *both below* the temperature of coincidence.

First, suppose the chain to be extended on the ground when the thermometer is at n° so that $\overline{54-n^\circ}$ shall express the number of degrees below the temperature of coincidence. Then, if that space be measured by the brass standard at 54° of temperature, it will be equal

It will appear, that there is a great difference in the above observed angles of the star with the N. end

equal $100 + \frac{\overline{54-n^\circ} \times ,00763}{12}$ feet; for the steel being contracted will evidently extend over a shorter space than it did at 54° by the quantity $\frac{\overline{54-n^\circ} \times ,00763}{12}$ feet.

Next, suppose the brass standard to be reduced to n° or $\overline{54-n^\circ}$ below the temperature of coincidence. Then, had the expansion of brass and steel been the same, the space $100 - \frac{\overline{54-n^\circ} \times ,00763}{12}$ feet, would now increase to $100 - \frac{\overline{54-n^\circ} \times ,00763}{12} + \frac{\overline{54-n^\circ} \times ,00763}{12}$ equal 100 feet by the brass scale, since that scale has contracted $\frac{\overline{54-n^\circ} \times ,00763}{12}$ feet in 100 feet.

But 100 feet of brass will have contracted $\frac{\overline{54-n^\circ} \times ,01237}{12}$ feet, and therefore the space in brass measure will be expressed by $100 - \frac{\overline{54-n^\circ} \times ,00763}{12} + \frac{\overline{54-n^\circ} \times ,01237}{12} = 100 + \frac{\overline{54-n^\circ} \times ,01237 - \overline{54-n}^\circ \times ,00763}{12}$ feet, when the standard temperature is n°. But if the standard temperature be s° then the space will measure

$$100 + \frac{\overline{54-s^\circ} \times ,01237 - \overline{54-n^\circ} \times ,00763}{12}$$ feet, when measured by the brass scale at s° of temperature.

Case 3d. Let the temperature of coincidence be *between* the standard temperature and the temperature of measurement.

1. Let the temperature of coincidence be 54° as before, and let the standard temperature be below 54°, so that $\overline{54-s^\circ}$ shall express the number of degrees below 54 for the reduction, and let n° be above 54, so that $\overline{n-54^\circ}$ expresses the excess of the temperature of measurement above that of coincidence, and $\overline{n-s^\circ}$ the excess of the temperature of measurement above the standard temperature.

Now,

end of the base; but that arose from the unfavourable weather in the mornings, at which time the telescope

Now, by Case 1st, the space over which the chain extends on the ground will be $100 + \frac{\overline{n-54^\circ} \times ,00763}{12}$ feet, compared with the brass scale at 54°. Had the contraction of brass been the same as that of steel, $100 + \frac{\overline{n-s^\circ} \times ,00763}{12}$ feet, would be the measure, by the brass scale at $\overline{54-s}^\circ$ below the temperature of coincidence. But it has contracted more by $\overline{54-s^\circ} + \frac{,01237 - ,00763}{12}$ feet in 100 feet; and consequently the space which the chain extends over, at n° of temperature, will, at $\overline{n-s^\circ}$ of temperature, measure, by the brass scale, $100 + \frac{\overline{n-s^\circ} \times ,00763 + \overline{54-s^\circ} \times \overline{,01237-,00763}}{12}$ feet.

2. Let the standard temperature be above 54°, and the temperature of measurement below it.

Then, by Case 2d, the space over which the chain extends, is $= 100 - \frac{\overline{54-n^\circ} \times ,00763}{12}$ feet, measured by the brass scale at 54°.—And $100 - \frac{\overline{s-n^\circ} \times ,00763}{12}$ feet would have been the measure at s° by the brass, had the expansion of steel and brass been equal. But the expansion of brass is more by $\overline{s-54^\circ} + \frac{\overline{,01237 - ,00763}}{12}$ feet. And therefore, if the space over which the steel chain extended, when the temperature was $\overline{54-n}^\circ$ below the temperature of coincidence, be measured by the brass standard, when the temperature is $\overline{s-54^\circ}$ above that of coincidence, the value of that space, in brass measure, will be $100 - \frac{(\overline{s-n^\circ} \times ,00763 + \overline{s-54^\circ} \times \overline{,0123-0076})}{12}$.

Hence, universally, if s° and n° denote as above, and t° temperature of coincidence, and S = the space on the ground over which the steel chain telescope of the circular instrument was directed to the flag staff.—It was intended to determine this angle, by having a blue light at the opposite end of the base, at the time that the star was at its greatest elongation; but, unfortunately, the weather became so unfavourable, that the star never made its appearance, for upwards of a fortnight—and as I was ready to move during all that time, I therefore determined to remain no longer at that station, but wait the event of more settled weather, which probably would happen before I had extended my operations very far, either to the eastward or westward of *Bangalore*. I therefore prepared to take angles at the most suitable places, and proceed to lay down the positions of the principal objects within the vicinity of *Bangalore*.

THE latitude of the South end of the base was obtained some time after, by observing, at a station North of *Bangalore*, which, with the two extremities of the base, formed a triangle. Those observations were made with the zenith sector on the 19th, 20th, and 21st of January, by taking the zenith distance of the star Aldebaran, whose declination was corrected

chain (whose length is 100 feet at t° of temperature) extends when the thermometer is at n°.—Then the formulæ for the different cases will be

$$1\ S = 100 + \frac{\overline{n - t}^\circ \times ,00763 - (s - t)^\circ \times ,01237}{12}$$

$$2\ S = 100 + \frac{\overline{t - s}^\circ \times ,01237 - (t - n)^\circ \times ,00763}{12}$$

$$3\ S = \begin{cases} \text{1st. } 100 + \dfrac{\overline{n - s}^\circ \times ,00763 + \overline{t - s}^\circ \times ,01237 - ,00763}{12} \\ \text{2d. } 100 - \left(\dfrac{\overline{s - n}^\circ \times ,06763 + \overline{s - t}^\circ \times ,01237 - ,00763}{12} \right) \end{cases} \quad \text{Feet,}$$

If the chain should measure + or — any quantity (q) at the temperature t° from wear &c. then put $100 \pm q$ in place of 100 in each equation.

corrected for precession, nutation, and aberration, for those days—and, in order to correct the error of collimation of the telescope, the instrument was turned upon its vertical axis on the 21st, and the zenith distance taken on the opposite part of the arc. —The latitude determined by the observation made on the 19th was 13°. 00′. 59,35″, and by that on the 20th, 13°. 00′. 58,72″. N. On the 21st, when the sector was turned, the latitude was observed 13°.00′ 22,6″, which will therefore give the mean 13°. 00′. 40,6″ N. From these it will appear that the error of collimation was 18,095″.

The latitude of that station being obtained, and also its distance from the south end of the base;—from knowing the angle which that distance made with the meridian, the distance on the meridian, between the station, and the point where a line falling from the southern extremity would cut it at right angles, was easily had, and the difference of latitude of the station and *that* point was computed, by allowing 60191 fathoms to the degree in latitude 13°. —And that gave 12°. 54′. 6,6″ for the latitude of the point of intersection on the meridian of the station.

The perpendicular, falling from the south end of the base on the meridian, was then converted into minutes and seconds, by allowing 60957 fathoms (b) for the degree on a great circle perpendicular to the meridian, and from that and the co-latitude of the point of intersection, the latitude of the southern extremity of the base was determined to be 12°. 54′.

(b) These measures have been determined by computing on the ellipsoid given by Col. Williams and Capt. Mudge, as resulting from their measurement of a degree perpendicular to the meridian in latitude 50° 41′ N. and of a degree on the meridian in the same latitude, as obtained from the measured arc between *Greenwich* and *Paris*.—The ratio of the diameters of that ellipsoid is nearly as 230 to 23,155. —The principles on which these computations are founded, with the most useful propositions relative to the ellipsoid, will be given hereafter, when the figure of the earth becomes the subject of investigation.

54′. 6,4″. In these distances, I did not compute on the chords of the arcs, because the instrument I had in use was not sufficient for that purpose.

Experiments for determining the Expansion of the Chain.

In making allowance for the expansion of the chain, in the annexed table, it will appear that I have differed both from General Roy and Colonel Williams. It may therefore be necessary to give the following account of the experiments which were made for ascertaining that allowance,—which experiments were made by the chain itself, observing its length at sun-rise and at one o'clock, between which hours the base was generally measured.

After the chain was extended in the coffers, in the manner formerly mentioned, it was carefully adjusted, at each end, to some particular marks on the register heads, about the hours of sun-rise. The finger screw of one of these brass sliders had been previously graduated into eight equal parts, on its circumference, which were counted, on its being turned, by another mark on the end of the slider, touching that part of the circumference. This finger screw was observed to make 26 revolutions in one inch, so that one of the divisions, on the circumference, was equal $\frac{1}{208}$ part of an inch. Things being thus adjusted, the experiments were made in the following order, and the mean temperature taken from three of the best thermometers I had, which remained the whole time in the coffers, with the chain; and these coffers were covered, in the same manner as they had been during the operations of the measurement.

December 11th, at one P. M. the temp[illegible] was 95°.

December 12th, at sever[illegible]

perature was 58°, therefore 37° is the difference, or fall of the thermometer, since the preceding day.

The chain had contracted 58 divisions on the micrometer screw, each of which being equal $\frac{1}{208}$ inches, therefore the whole expansion of the chain was $\frac{58}{208}$ = ,27884 inches—and this divided by 37° gives ,00721 inches, the expansion of the chain due to one degree of the thermometer.

December 13th, at half past six A. M. the mean of three thermometers was 56° which was 39° decrease of temperature since the preceding day at one o'clock P. M.—The chain had contracted 60 divisions — therefore $\frac{60}{208}$ divided by 39° = ,007396 inches.

At one P. M. the same day, the temperature was 97°, and consequently the increase since morning was 41°. The chain had expanded 63 divisions, hence $\frac{63}{208}$ divided by 41° gives ,0073853 inches.

December 15th.—At seven A. M. the temperature was 62°, and at one P. M. 93°—and therefore the increase since morning was 31°. The chain had expanded 46 divisions, therefore $\frac{46}{208}$ divided by 31° = ,00713 inches.

December 16th, at half past six A. M. the temperature was 51°,2 which was 41°,8 below the preceding day at one o'clock P. M. The chain had contracted 59 divisions, which proceeding as before, gives ,006786 inches.

December 17th, at half past six A. M. the temperature was 56°, and at one P. M. it was 92°—whose difference is 36°. The chain had 58 divisions, which will give ,00761 inches.

The mean of all these being ,007253 inches, I have therefore made the expansion of the chain due to 1° of temperature above 62° to be ,0073 inches.

XI. On